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A DECOMPOSITION METHOD FOR ESTIMATING MODAL PARAMETERS

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1 Introduction

This paper presents a method of decomposing free-response ensemble data into modal components, while enabling frequency, damping, and mode-shape estimation for generally damped linear multi-degree-of-freedom systems. The method is based on the state-variable model of a vibration system

The equations of motion for free vibrations are $\mathbf{M}\ddot{\mathbf{x}}+\mathbf{C}\dot{\mathbf{x}}+\mathbf{K}\mathbf{x}=\mathbf{0}$, where $\mathbf{x}$ is an $n\times 1$ array of mass displacements, and $\mathbf{M},\mathbf{C}$, and $\mathbf{K}$, are the $n\times n$ mass, damping, and stiffness matrices. Defining a $2n\times 1$ state vector $\mathbf{y}^T=[\dot{\mathbf{x}}^T,\mathbf{x}^T]$, and introducing $\mathbf{M}\dot{\mathbf{x}}-\mathbf{M}\dot{\mathbf{x}}=\mathbf{0}$, yields unforced equations of motion of the form $\mathbf{A}\dot{\mathbf{y}}+\mathbf{B}\mathbf{y}=\mathbf{0}$, where $\mathbf{A}$ consists of $\mathbf{0}$ and $\mathbf{C}$ on the block diagonals, and $\mathbf{M}$ on the off-diagonal blocks, and $\mathbf{B}$ is block diagonal with $-\mathbf{M}$ and $\mathbf{K}$. Assuming a response of the form $\mathbf{y}=e^{\alpha t}\underline{\phi}$, the complex modes are obtained from the eigenvalue problem

$$\alpha\mathbf{A}\underline{\phi}+\mathbf{B}\underline{\phi}=\mathbf{0}, \qquad (1)$$

which hass complex eigenvalues α and eigenvectors $\underline{\phi}=[\mathbf{v}^T,\mathbf{w}^T]^T$, where the $\mathbf{w}$ correspond to displacement modes. Real and imaginary parts of α quantify the modal frequency and damping. Our decomposition is aimed at estimating ϕ and α.

2 Decomposition Strategy and Example

The decomposition uses the free-response state-variable ensemble $\mathbf{Y}=[\mathbf{V}^T,\mathbf{X}^T]^T$, where n-mass by N-sample $\mathbf{X}$ is a displacement ensemble, and $\mathbf{V}=\mathbf{X}\mathbf{D}^T\approx\dot{\mathbf{X}}$ is an approximate velocity ensemble made from an $(N-2)\times N$ matrix $\mathbf{D}$ of centered finite differences. Thus $\mathbf{V}$ is $n\times(N-2)$, and so the first and last columns of $\mathbf{X}$ are dropped so that $\mathbf{Y}$ has compatible partitions.

We then take the derivative $\mathbf{W}=\mathbf{Y}\mathbf{D}^T\approx\dot{\mathbf{Y}}$, this time using an $(N-4)\times(N-2)$ difference matrix $\mathbf{D}$. The first and last time samples of $\mathbf{Y}$ are then dropped so that the dimensions of $\mathbf{Y}$ and $\mathbf{W}$ are the same. We form a correlation matrix $\mathbf{R}=\mathbf{Y}\mathbf{Y}^T/N$ and a nonsymmetric matrix $\mathbf{N}=\mathbf{Y}\mathbf{W}^T/N$.

The eigenvalue problem is then

$$\alpha\mathbf{R}\underline{\psi}=\mathbf{N}\underline{\psi}. \qquad (2)$$

The eigenvalues of (2) approximate the state-variable eigenvalues, containing information about damping and frequency. The inverse of the modal matrix from equation (2) resembles the complex linear normal modal matrix of equation (1).

As an example, we simulate a chain of eight unit masses connected by unit springs, with a dashpot grounded to the eighth mass, with 10-bit noise. Fig. 1 shows the modal frequencies and damping from (1) and (2), and the third mode comparison.

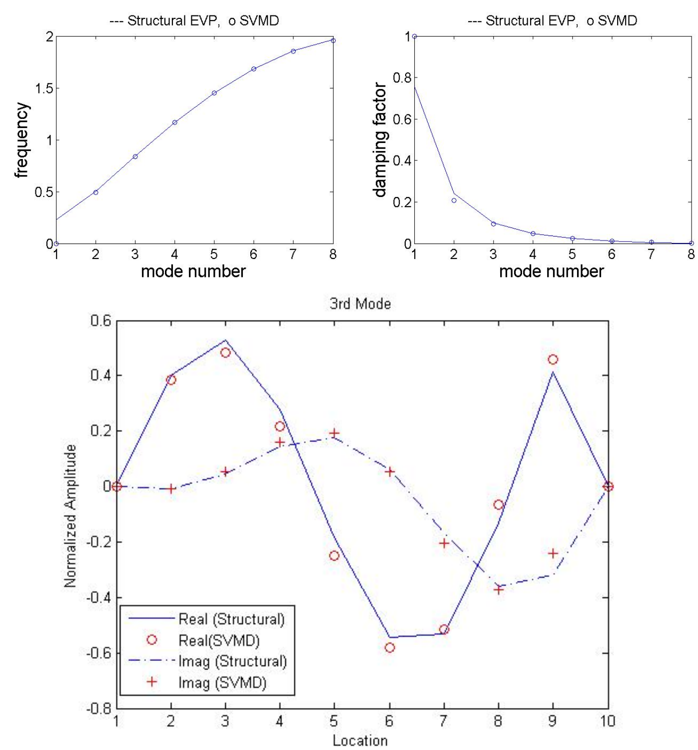

Figure 1. Modal frequency and damping (top) from equations (1) and (2). Third mode from equations (1) and (2).